Chapter 14 Oscillations

Problems 10, 17, 29, 31, 43, 49, 62, 67 Bonus: 2, 6, 16, 32, 64

2 If the amplitude of a simple harmonic oscillator is tripled, by what factor is the energy changed?

Determine the Concept The energy of a simple harmonic oscillator varies as the square of the amplitude of its motion. Hence, tripling the amplitude increases the energy by a factor of 9.

6 Two systems each consist of a spring with one end attached to a block and the other end attached to a wall. The springs are horizontal, and the blocks are supported from below by a frictionless horizontal table. The blocks are oscillating in simple harmonic motions with equal amplitudes. However, the force constant of spring A is four times as large as the force constant of spring B. How do their maximum speeds compare? (*a*) $v_{\text{A max}} = v_{\text{B max}}$, (*b*) $v_{\text{A max}} = 2v_{\text{B max}}$, (*c*) $v_{\text{A max}} = 4v_{\text{B max}}$, (*d*) This comparison cannot be done by using the data given.

Determine the Concept The maximum speed of a simple harmonic oscillator is the product of its angular frequency and its amplitude. The angular frequency of a simple harmonic oscillator is the square root of the quotient of the force constant of the spring and the mass of the oscillator.

Relate the maximum speed of system A to its force constant:

$$v_{\text{A max}} = \omega_{\text{A}} A_{\text{A}} = \sqrt{\frac{k_{\text{A}}}{m_{\text{A}}}} A_{\text{A}}$$

Relate the maximum speed of system B to its force constant:

$$v_{\text{B max}} = \omega_{\text{B}} A_{\text{B}} = \sqrt{\frac{k_{\text{B}}}{m_{\text{B}}}} A_{\text{B}}$$

Divide the first of these equations by the second and simplify to obtain:

$$\frac{v_{\text{A max}}}{v_{\text{B max}}} = \frac{\sqrt{\frac{k_{\text{A}}}{m_{\text{A}}}} A_{\text{A}}}{\sqrt{\frac{k_{\text{B}}}{m_{\text{B}}}} A_{\text{B}}} = \sqrt{\frac{m_{\text{B}}}{m_{\text{A}}} \frac{k_{\text{A}}}{k_{\text{B}}}} \frac{A_{\text{A}}}{A_{\text{B}}}$$

Because the systems differ only in their force constants:

$$\frac{v_{\text{A max}}}{v_{\text{B max}}} = \sqrt{\frac{k_{\text{A}}}{k_{\text{B}}}}$$

Substituting for k_{A} and simplifying yields:

$$\frac{v_{\text{A max}}}{v_{\text{B max}}} = \sqrt{\frac{4k_{\text{B}}}{k_{\text{B}}}} = 2 \Rightarrow v_{\text{A max}} = 2v_{\text{B max}}$$

$\boxed{(b)}$ is correct.

10 Two mass–spring systems oscillate with periods T_A and T_B. If $T_A = 2T_B$ and the systems' springs have identical force constants, it follows that the systems' masses are related by (*a*) $m_A = 4m_B$, (*b*) $m_A = m_B/\sqrt{2}$, (*c*) $m_A = m_B/2$, (*d*) $m_A = m_B/4$.

Picture the Problem We can use $T = 2\pi\sqrt{m/k}$ to express the periods of the two mass-spring systems in terms of their force constants. Dividing one of the equations by the other will allow us to express m_A in terms of m_B.

Express the period of system A:

$$T_A = 2\pi\sqrt{\frac{m_A}{k_A}} \Rightarrow m_A = \frac{k_A T_A^2}{4\pi^2}$$

Relate the mass of system B to its period:

$$m_B = \frac{k_B T_B^2}{4\pi^2}$$

Divide the first of these equations by the second and simplify to obtain:

$$\frac{m_A}{m_B} = \frac{\frac{k_A T_A^2}{4\pi^2}}{\frac{k_B T_B^2}{4\pi^2}} = \frac{k_A T_A^2}{k_B T_B^2}$$

Because the force constants of the two systems are the same:

$$\frac{m_A}{m_B} = \frac{T_A^2}{T_B^2} = \left(\frac{T_A}{T_B}\right)^2$$

Substituting for T_A and simplifying yields:

$$\frac{m_A}{m_B} = \left(\frac{2T_B}{T_B}\right)^2 = 4 \Rightarrow m_A = 4m_B$$

$\boxed{(a)}$ is correct.

16 Two simple pendulums are related as follows. Pendulum A has a length L_A and a bob of mass m_A; pendulum B has a length L_B and a bob of mass m_B. If the period of A is twice that of B, then (*a*) $L_A = 2L_B$ and $m_A = 2m_B$, (*b*) $L_A = 4L_B$ and $m_A = m_B$, (*c*) $L_A = 4L_B$ whatever the ratio m_A/m_B, (*d*) $L_A = \sqrt{2}L_B$ whatever the ratio m_A/m_B.

Picture the Problem The period of a simple pendulum is independent of the mass of its bob and is given by $T = 2\pi\sqrt{L/g}$.

Express the period of pendulum A:

$$T_A = 2\pi\sqrt{\frac{L_A}{g}}$$

Express the period of pendulum B:

$$T_B = 2\pi\sqrt{\frac{L_B}{g}}$$

Divide the first of these equations by the second and solve for L_A/L_B:

$$\frac{L_A}{L_B} = \left(\frac{T_A}{T_B}\right)^2$$

Substitute for T_A and solve for L_B to obtain:

$$L_A = \left(\frac{2T_B}{T_B}\right)^2 L_B = 4L_B$$

$\boxed{(c)}$ is correct.

17 Two simple pendulums are related as follows. Pendulum A has a length L_A and a bob of mass m_A; pendulum B has a length L_B and a bob of mass m_B. If the frequency of A is one-third that of B, then (*a*) $L_A = 3L_B$ and $m_A = 3m_B$, (*b*) $L_A = 9L_B$ and $m_A = m_B$, (*c*) $L_A = 9L_B$ regardless of the ratio m_A/m_B, (*d*) $L_A = \sqrt{3}L_B$ regardless of the ratio m_A/m_B.

Picture the Problem The frequency of a simple pendulum is independent of the mass of its bob and is given by $f = \frac{1}{2\pi}\sqrt{g/L}$.

Express the frequency of pendulum A:

$$f_A = \frac{1}{2\pi}\sqrt{\frac{g}{L_A}} \Rightarrow L_A = \frac{g}{4\pi^2 f_A^2}$$

Similarly, the length of pendulum B is given by:

$$L_B = \frac{g}{4\pi^2 f_B^2}$$

Divide the first of these equations by the second and simplify to obtain:

$$\frac{L_A}{L_B} = \frac{\frac{g}{4\pi^2 f_A^2}}{\frac{g}{4\pi^2 f_B^2}} = \frac{f_B^2}{f_A^2} = \left(\frac{f_B}{f_A}\right)^2$$

Substitute for f_A to obtain:

$$\frac{L_A}{L_B} = \left(\frac{f_B}{\frac{1}{3}f_B}\right)^2 = 9 \Rightarrow L_A = 9L_B$$

$\boxed{(c)}$ is correct.

29 The position of a particle is given by $x = (7.0\text{ cm})\cos 6\pi t$, where t is in seconds. What are (*a*) the frequency, (*b*) the period, and (*c*) the amplitude of the particle's motion? (*d*) What is the first time after $t = 0$ that the particle is at its equilibrium position? In what direction is it moving at that time?

Picture the Problem The position of the particle is given by $x = A\cos(\omega t + \delta)$ where A is the amplitude of the motion, ω is the angular frequency, and δ is a phase constant. The frequency of the motion is given by $f = \omega/2\pi$ and the period of the motion is the reciprocal of its frequency.

(*a*) Use the definition of ω to determine f:

$$f = \frac{\omega}{2\pi} = \frac{6\pi\text{ s}^{-1}}{2\pi} = \boxed{3.00\text{ Hz}}$$

(*b*) Evaluate the reciprocal of the frequency:

$$T = \frac{1}{f} = \frac{1}{3.00\text{ Hz}} = \boxed{0.333\text{ s}}$$

(*c*) Compare $x = (7.0\text{ cm})\cos 6\pi t$ to $x = A\cos(\omega t+\delta)$ to conclude that:

$$A = \boxed{7.0\,\text{cm}}$$

(*d*) Express the condition that must be satisfied when the particle is at its equilibrium position:

$$\cos\omega t = 0 \Rightarrow \omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{2\omega}$$

Substituting for ω yields:

$$t = \frac{\pi}{2(6\pi)} = \boxed{0.0833\,\text{s}}$$

Differentiate x to find $v(t)$:

$$v = \frac{d}{dt}\left[(7.0\,\text{cm})\cos 6\pi t\right] = -(42\pi\text{ cm/s})\sin 6\pi t$$

Evaluate $v(0.0833\text{ s})$:

$$v(0.0833\,\text{s}) = -(42\pi\text{ cm/s})\sin 6\pi(0.0833\,\text{s}) < 0$$

Because $v < 0$, the particle is moving in the negative direction at $t = 0.0833$ s.

31 A particle of mass *m* begins at rest from $x = +25$ cm and oscillates about its equilibrium position at $x = 0$ with a period of 1.5 s. Write expressions for (*a*) the position *x* as a function of *t,* (*b*) the velocity v_x as a function of *t,* and (*c*) the acceleration a_x as a function of *t.*

Picture the Problem The position of the particle as a function of time is given by $x = A\cos(\omega t+\delta)$. Its velocity as a function of time is $v_x = -A\omega\sin(\omega t+\delta)$ and its acceleration is $a_x = -A\omega^2\cos(\omega t+\delta)$. The initial position and velocity give us two equations from which to determine the amplitude *A* and phase constant δ.

(*a*) Express the position, velocity, and acceleration of the particle as a function of *t*:

$$x = A\cos(\omega t+\delta) \quad (1)$$

$$v_x = -A\omega\sin(\omega t+\delta) \quad (2)$$

$$a_x = -A\omega^2\cos(\omega t+\delta) \quad (3)$$

Find the angular frequency of the particle's motion:

$$\omega = \frac{2\pi}{T} = \frac{4\pi}{3}\,\text{s}^{-1} = 4.19\,\text{s}^{-1}$$

Relate the initial position and velocity to the amplitude and phase constant:

$$x_0 = A\cos\delta$$

and

$$v_0 = -\omega A\sin\delta$$

Divide the equation for v_0 by the equation for x_0 to eliminate *A*:

$$\frac{v_0}{x_0} = \frac{-\omega A\sin\delta}{A\cos\delta} = -\omega\tan\delta$$

Solving for δ yields:

$$\delta = \tan^{-1}\left(-\frac{v_0}{x_0\omega}\right) = \tan^{-1}\left(-\frac{0}{x_0\omega}\right) = 0$$

Substitute in equation (1) to obtain:

$$x = (25\,\text{cm})\cos\left[\left(\frac{4\pi}{3}\text{s}^{-1}\right)t\right] = \boxed{(0.25\,\text{m})\cos\left[\left(4.2\,\text{s}^{-1}\right)t\right]}$$

(b) Substitute in equation (2) to obtain:

$$v_x = -(25\,\text{cm})\left(\frac{4\pi}{3}\,\text{s}^{-1}\right)\sin\left[\left(\frac{4\pi}{3}\,\text{s}^{-1}\right)t\right]$$
$$= \boxed{-(1.0\,\text{m/s})\sin\left[\left(4.2\,\text{s}^{-1}\right)t\right]}$$

(c) Substitute in equation (3) to obtain:

$$a_x = -(25\,\text{cm})\left(\frac{4\pi}{3}\,\text{s}^{-1}\right)^2\cos\left[\left(\frac{4\pi}{3}\,\text{s}^{-1}\right)t\right]$$
$$= \boxed{-\left(4.4\,\text{m/s}^2\right)\cos\left[\left(4.2\,\text{s}^{-1}\right)t\right]}$$

32 Find (a) the maximum speed and (b) the maximum acceleration of the particle in Problem 31. (c) What is the first time that the particle is at $x = 0$ and moving to the right?

Picture the Problem The maximum speed and maximum acceleration of the particle in are given by $v_{\text{max}} = A\omega$ and $a_{\text{max}} = A\omega^2$. The particle's position is given by $x = A\cos(\omega t + \delta)$ where A = 7.0 cm, $\omega = 6\pi\ \text{s}^{-1}$, and $\delta = 0$, and its velocity is given by $v = -A\omega\sin(\omega t + \delta)$.

(a) Express v_{max} in terms of A and ω:

$$v_{\text{max}} = A\omega = (7.0\,\text{cm})\left(6\pi\,\text{s}^{-1}\right) = 42\pi\,\text{cm/s}$$
$$= \boxed{1.3\,\text{m/s}}$$

(b) Express a_{max} in terms of A and ω:

$$a_{\text{max}} = A\omega^2 = (7.0\,\text{cm})\left(6\pi\,\text{s}^{-1}\right)^2$$
$$= 252\pi^2\,\text{cm/s}^2 = \boxed{25\,\text{m/s}^2}$$

(c) When $x = 0$:

$$\cos\omega t = 0 \Rightarrow \omega t = \cos^{-1}(0) = \frac{\pi}{2}, \frac{3\pi}{2}$$

Evaluate v for $\omega t = \frac{\pi}{2}$:

$$v = -A\omega\sin\left(\frac{\pi}{2}\right) = -A\omega$$

That is, the particle is moving to the left.

Evaluate v for $\omega t = \frac{3\pi}{2}$:

$$v = -A\omega\sin\left(\frac{3\pi}{2}\right) = A\omega$$

That is, the particle is moving to the right.

Solve $\omega t = \frac{3\pi}{2}$ for t to obtain:

$$t = \frac{3\pi}{2\omega} = \frac{3\pi}{2\left(6\pi\,\text{s}^{-1}\right)} = \boxed{0.25\,\text{s}}$$

43 A 1.50-kg object on a frictionless horizontal surface oscillates at the end of a spring of force constant k = 500 N/m. The object's maximum speed is 70.0 cm/s. (a) What is the system's total mechanical energy? (b) What is the amplitude of the motion?

Picture the Problem The total mechanical energy of the oscillating object can be expressed in terms of its kinetic energy as it passes through its equilibrium position: $E_{tot} = \frac{1}{2}mv_{max}^2$. Its total energy is also given by $E_{tot} = \frac{1}{2}kA^2$. We can equate these expressions to obtain an expression for A.

(*a*) Express the total mechanical energy of the object in terms of its maximum kinetic energy:

$$E = \tfrac{1}{2}mv_{max}^2$$

Substitute numerical values and evaluate E:

$$E = \tfrac{1}{2}(1.50\,\text{kg})(0.700\,\text{m/s})^2 = 0.3675\,\text{J} = \boxed{0.368\,\text{J}}$$

(*b*) Express the total mechanical energy of the object in terms of the amplitude of its motion:

$$E_{tot} = \tfrac{1}{2}kA^2 \Rightarrow A = \sqrt{\frac{2E_{tot}}{k}}$$

Substitute numerical values and evaluate A:

$$A = \sqrt{\frac{2(0.3675\,\text{J})}{500\,\text{N/m}}} = \boxed{3.83\,\text{cm}}$$

49 A 3.0-kg object on a frictionless horizontal surface is attached to one end of a horizontal spring, oscillates with an amplitude of 10 cm and a frequency of 2.4 Hz. (*a*) What is the force constant of the spring? (*b*) What is the period of the motion? (*c*) What is the maximum speed of the object? (*d*) What is the maximum acceleration of the object?

Picture the Problem (*a*) The angular frequency of the motion is related to the force constant of the spring through $\omega^2 = k/m$. (*b*) The period of the motion is the reciprocal of its frequency. (*c*) and (*d*) The maximum speed and acceleration of an object executing simple harmonic motion are $v_{max} = A\omega$ and $a_{max} = A\omega^2$, respectively.

(*a*) Relate the angular frequency of the motion to the force constant of the spring:

$$\omega^2 = \frac{k}{m} \Rightarrow k = m\omega^2 = 4\pi^2 f^2 m$$

Substitute numerical values to obtain:

$$k = 4\pi^2(2.4\,\text{s}^{-1})^2(3.0\,\text{kg}) = 682\,\text{N/m} = \boxed{0.68\,\text{kN/m}}$$

(*b*) Relate the period of the motion to its frequency:

$$T = \frac{1}{f} = \frac{1}{2.4\,\text{s}^{-1}} = 0.417\,\text{s} = \boxed{0.42\,\text{s}}$$

(*c*) The maximum speed of the object is given by:

$$v_{max} = A\omega = 2\pi f A$$

Substitute numerical values and evaluate v_{max}:

$$v_{max} = 2\pi(2.4\,\text{s}^{-1})(0.10\,\text{m}) = 1.51\,\text{m/s} = \boxed{1.5\,\text{m/s}}$$

(*d*) The maximum acceleration of the object is given by:

$$a_{max} = A\omega^2 = 4\pi^2 f^2 A$$

Substitute numerical values and evaluate a_{max}:

$$a_{max} = 4\pi^2 (2.4\,\text{s}^{-1})^2 (0.10\,\text{m}) = \boxed{23\,\text{m/s}^2}$$

62 If the period of a 70.0-cm-long simple pendulum is 1.68 s, what is the value of *g* at the location of the pendulum?

Picture the Problem We can find the value of *g* at the location of the pendulum by solving the equation $T = 2\pi\sqrt{L/g}$ for *g* and evaluating it for the given length and period.

Express the period of a simple pendulum where the gravitational field is *g*:

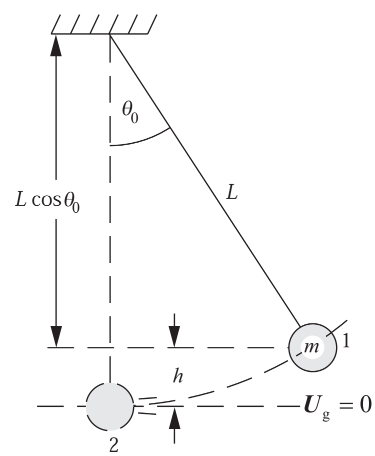
$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 L}{T^2}$$

Substitute numerical values and evaluate *g*:

$$g = \frac{4\pi^2 (0.700\,\text{m})}{(1.68\,\text{s})^2} = \boxed{9.79\,\text{m/s}^2}$$

64 Show that the total energy of a simple pendulum undergoing oscillations of small amplitude ϕ_0 (in radians) is $E \approx \frac{1}{2} mgL\phi_0^2$. *Hint: Use the approximation* $\cos\phi \approx 1 - \frac{1}{2}\phi^2$ *for small* ϕ.

Picture the Problem The figure shows the simple pendulum at maximum angular displacement ϕ_0. The total energy of the simple pendulum is equal to its initial gravitational potential energy. We can apply the definition of gravitational potential energy and use the small-angle approximation to show that $E \approx \frac{1}{2} mgL\phi_0^2$.



Express the total energy of the simple pendulum at maximum displacement:

$$E = U_{\text{max displacement}} = mgh$$

Referring to the diagram, express *h* in terms of *L* and ϕ_0:

$$h = L - L\cos\phi_0 = L(1 - \cos\phi_0)$$

Substituting for *h* yields:

$$E = mgL[1 - \cos\phi_0]$$

From the power series expansion for $\cos\phi$, for $\phi << 1$:

$$\cos\phi \approx 1 - \tfrac{1}{2}\phi^2$$

Substitute and simplify to obtain:

$$E = mgL\left[1 - \left(1 - \tfrac{1}{2}\phi_0^2\right)\right] = \boxed{\tfrac{1}{2}mgL\phi_0^2}$$

67 A thin 5.0-kg disk with a 20-cm radius is free to rotate about a fixed horizontal axis perpendicular to the disk and passing through its rim. The disk is displaced slightly from equilibrium and released. Find the period of the subsequent simple harmonic motion.

Picture the Problem The period of this physical pendulum is given by $T = 2\pi\sqrt{I/MgD}$ where I is the moment of inertia of the thin disk about the fixed horizontal axis passing through its rim. We can use the parallel-axis theorem to express I in terms of the moment of inertia of the disk about an axis through its center of mass and the distance from its center of mass to its pivot point.

Express the period of a physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{MgD}}$$

Using the parallel-axis theorem, find the moment of inertia of the thin disk about an axis through the pivot point:

$$I = I_{\text{cm}} + MR^2 = \tfrac{1}{2}MR^2 + MR^2 = \tfrac{3}{2}MR^2$$

Substituting for I and simplifying yields:

$$T = 2\pi\sqrt{\frac{\tfrac{3}{2}MR^2}{MgR}} = 2\pi\sqrt{\frac{3R}{2g}}$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{\frac{3(0.20\,\text{m})}{2(9.81\,\text{m/s}^2)}} = \boxed{1.1\,\text{s}}$$